Notes from the HW 3-1 due 2009-09-09

§ 3.1 #7

$$y = \frac{1}{2}x^2;\ x_0 = 3;\ x_1 = 4$$

(a) Find the average rate of change of *y* with respect to *x* over the interval $[x_0, x_1]$.

Ans

$$\text{Average rate of change} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{.5(4)^2 - .5(3)^2}{4-3} = 7/2$$

Note – this is definition 3.1.3 on page 173.

(b) Find the instantaneous rate of change of *y* with respect to *x* at the given value of x_0,

Ans

Instantaneous rate of change =

$$\lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x \to 3} \frac{.5x^2 - .5(3)^2}{(x-3)} = (.5)\lim_{x \to 3} \frac{x^2 - 9}{x-3} = (.5)\lim_{x \to 3}(x+3) = 3$$

Note – this is definition 3.1.4 on page 173. Also – the x_1 in definition 3.1.4 is **not** the same as the one in the problem. Poor choice of notation.

(c) Find the instantaneous rate of change of *y* with respect to *x* at a general *x*-value x_0.

Ans

Instantaneous rate of change (with x_0 **not** replaced with 3) =

$$\lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \lim_{x \to x_0} \frac{.5x^2 - .5(x_0)^2}{(x - x_0)} = (.5)\lim_{x \to x_0} \frac{x^2 - x_0^2}{x - x_0} = (.5)\lim_{x \to x_0}(x + x_0) = x_0$$

Note – the algebra now lets us find a rule for ANY *x*-value. Also, can you see how solving the problem once (or twice) with numbers teaches us what the algebra should look like in this more general step?

(d) Sketch the graph of $y = f(x)$ together with the secant line whose slope is given by the result in part (a), and indicate graphically the slope of the curve given by the result in part (b).

Ans

Can't graph this clearly …..

Graph a line segment connecting (3, 4.5) and (4, 8).

Graph the line you found in (b) …. The slope is *3*, and the point it goes through is (3, 4.5).

And, of course, graph the function.